3/1/2001 MA401 Complex Analysis Name:________________

Dr. Lunsford Test 1 (100 Points Total)

I. Perform the indicated computations. Write all answers in $a+bi$ form. (5 points each – 25 points total)

1. $\left|\dfrac{3i}{4-8i}\right|$

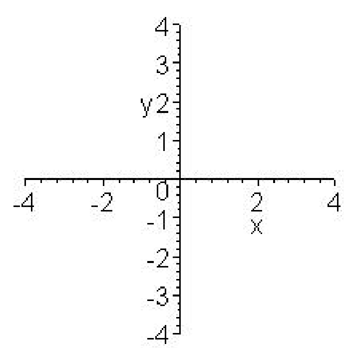
2. $\dfrac{2+i}{4-3i}$

3. $\cos(3+2i)$

4. $e^{1-\frac{5p}{4}i}$

5. $\log(2i)$ and $Log(2i)$

II. Convert the complex number $z=-3+4i$ to polar form. Graph the number on the axes provided showing z, $|z|$, and $\arg(z)$ on the graph. You should provide a calculator approximation for $\arg(z)$. (6 points)



III. Solve each of the following equations. I.e., find all $z \in \mathbb{C}$ such that the equation is true. (6 points each – 12 total)

1. $\sin z = i$

2. $e^z = 2i$

IV. Find the indicated limits. Neatly show all of your work. (4 each – 16 points total)

a. $\lim_{n\to\infty} z_n$ where $z_n = n\left(\frac{i}{2}\right)^n$

b. $\lim_{n\to\infty} z_n$ where $z_n = \frac{2n-1}{1-n} + \frac{1-2n^2}{3n^2} i$

c. $\lim_{z\to 0} \frac{\bar{z}}{z}$ (the limit as $z \to 0$ of z conjugate over z)

d. $\lim_{z\to 1+i} (z^2 - \text{Re}(z))$

V. Use the inequality $|z-1-i| \le |z|$ to answer the following. Neatly show all of your work and clearly indicate your answers to parts (b), (c) and (d). (16 points total)

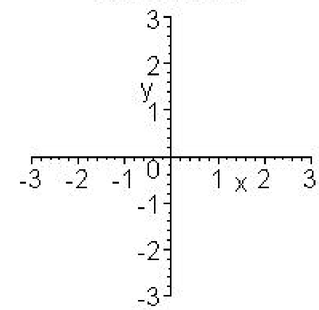
(a) Graph the set of all $z \in \mathbb{C}$ that satisfy the equation. Give the equations of any boundaries in rectangular coordinates. (6 points)

(b) Describe (via an equation or inequality) and graph the interior of the set and the boundary of the set on the axes provided. (4 points)

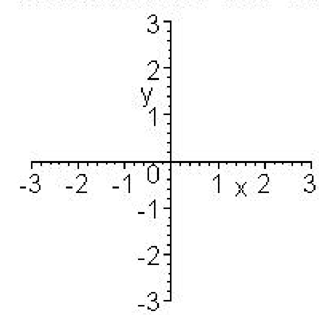
(c) Determine if the set is open and/or closed. Explain why. (3 points)

(d) Determine if the set is compact or not compact. Explain why. (3 points)

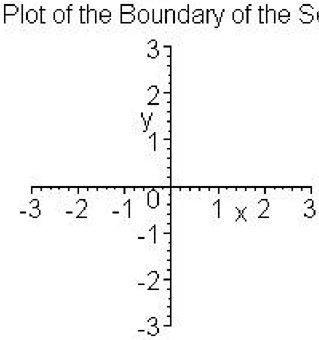
Plot of the Set



Plot of the Interior of the Set



Plot of the Boundary of the Set



VI. Determine the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(3+4i)^n}{2n}(z-i)^{2n}$.

Describe the open disk of convergence and graph it in the complex plane. (5 points)

VII. Given $f(z) = iz^2 + i$ please answer the following. (7 points total)

a. Find $f'(z)$. (2 points)

b. Show that the Cauchy Riemann Equations for f hold for all $z \in \mathbb{C}$. (5 points)

VIII. Given $f(z) = i|z|^2 - z$, find all $z \in \mathbb{C}$ where f is differentiable. (7 points)

IX. Work <u>one</u> of the following three problems. Clearly indicate which problem you are working for credit. (6 points)

1. Prove *de Moivre's Theorem*: If n is any integer, then

$$(\cos(q) + i\sin(q))^n = \cos(nq) + i\sin(nq)$$

2. Prove the *Parallelogram Equality*: For any complex numbers z and w the following holds:

$$|z+w|^2 + |z-w|^2 = 2(|z|^2 + |w|^2)$$

3. Recall that for $z \in \mathbb{C}$ we define

$$\sin z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} z^{2n+1} \text{ and } \cos z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} z^{2n}$$

   a. Show that the cosine function is defined for all $z \in \mathbb{C}$. (i.e. show that the radius of convergence of the power series for the cosine is infinite.)
   b. Using the above definitions, prove that $\frac{d}{dz}\cos z = -\sin z$

<u>BONUS</u>: Work another problem from IX. (5 points)