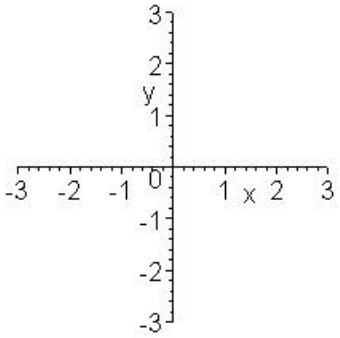
4/19/2001 MA401 Complex Analysis Name:_____
Dr. Lunsford Test 2 (100 Points Total)

I. Find all values of $(-4+4\sqrt{3}\,i)^{1/3}$ in $a+bi$ form (round $a$ and $b$ to two decimal places please). Graph these roots on the axes provided below. (10 points)

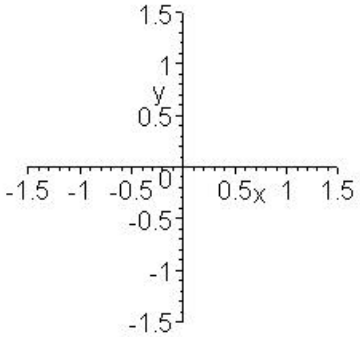



II. Given the complex function $f(z)=z\cos\left(\frac{1}{z}\right)$, please answer the following. (10 points total)

a. Find the Laurent series expansion for $f$ about zero. (7 points)

b. Indicate the values of $c_0$, $c_1$, and $\mathrm{Res}(f,0)$ from the Laurent series found in part (a) above. (3 points)

III. Without computing the values, plot the $6^{th}$ roots of unity on the complex plane below. Clearly identify the roots (you may call them $r_1$, $r_2$, etc.) and show the angle $q$ between the roots. (6 points)




IV. Given the function $f(z)=\dfrac{\sin(z)}{z^2(z-i)^2}$, please answer the following. (10 points total)

a. Find the order of the pole at $z=i$ (you must justify your answer). (5 points)

b. Find $\mathrm{Res}(f,0)$. (5 points)

V. Evaluate the integrals over the indicated curves. For each problem clearly indicate what theorem(s) you use and why you know you may use the theorem(s). Please draw pictures to help clarify your ideas!! (8 points each – 64 points total)

a. $\displaystyle\oint_{\Gamma}\frac{z^3}{z^2+1}dz$, where $\Gamma$ is the closed curve given by $|z-i|=1/4$.

b. $\int_{\Gamma} \frac{1}{z^2+2z+2} dz$, where $\Gamma$ is the closed curve given by $|z|=3$.

c. $\int_{\Gamma} \cos(6z) dz$, where $\Gamma$ is the straight line segment from $i$ to $-i$.

d. $\int_{\Gamma} \frac{z^2+i}{\sin z} dz$, where $\Gamma$ is the closed curve given by $|z|=1$.

e. $\int_{\Gamma} \cos\left(\frac{1}{z}\right) dz$, where $\Gamma$ is the closed curve given by $|z-1+2i|=1$.

f. $\int_{\Gamma} \left(\bar{z}\right)^2 dz$, where $\Gamma$ is the circle of radius 2 traversed once about the point $i$.

g. $\int \frac{\cos(z^2)}{(z-\sqrt{p}\, i)^3} dz$, where $\Gamma$ is the closed curve given by $|z| = 3$.

h. $\oint_{\Gamma} \frac{3+z^2}{(z+i)^2 z} dz$, where $\Gamma$ is the circle of radius 2 traversed once about the origin.