Pledge:

3/31/2010	MATH361 Calculus III	Name: Solution
Dr. Lunsford	Quiz 6	(40 Points Total)

Please show all work on this quiz including ANY substitutions you may make. There are two BONUS problems on this quiz. You will only get credit for one. Please clearly indicate which BONUS problem you want me to grade.

Problem I. Evaluate the integral $\int_0^1\int_0^z\int_0^y ze^{-y^2}\,dx\,dy\,dz$ (10 points)

$$\int_0^1\int_0^z xze^{-y^2}\Big|_{x=0}^{y} dy\,dz = \int_0^1\int_0^z yze^{-y^2}dy\,dz \qquad \begin{array}{l} u=y^2 \\ du=2y\,dy \end{array}$$

$$= \int_0^1\int_0^{z^2} \tfrac{1}{2}ze^{-u}\,du\,dz = \int_0^1 -\tfrac{1}{2}ze^{-u}\Big|_0^{z^2} dz$$

$$= \int_0^1 -\tfrac{1}{2}ze^{-z^2} + \tfrac{1}{2}z\,dz = \tfrac{1}{4}e^{-z^2} + \tfrac{1}{4}z^2\Big|_0^1 = \tfrac{1}{4}e^{-1} + \tfrac{1}{4} - \left(\tfrac{1}{4}\right)$$

$$\begin{array}{l} u=-z^2 \\ du=-2z\,dz \end{array} \qquad = \boxed{\tfrac{1}{4}e^{-1}}$$

Problem II. Set up (DO NOT INTEGRATE) an integral to find the surface area of the parametric surface $\mathbf{r}(u,v) = u\cos(v)\mathbf{i} + u\sin(v)\mathbf{j} + v\mathbf{k}$ where $0 \le u \le 2$ and $0 \le v \le u$. (10 points)

$r_u = \langle \cos v, \sin v, 0\rangle$

$r_v = \langle -u\sin u, u\cos v, 1\rangle$

$|r_u \times r_v| = |\langle \sin v, -\cos v, u\cos^2 v + u\sin^2 v\rangle|$

$$= |\langle \sin v, -\cos v, u\rangle| = \sqrt{\sin^2 v + \cos^2 v + u^2} = \sqrt{1+u^2}$$

$$\int_0^2\int_0^u \sqrt{1+u^2}\,dv\,du$$

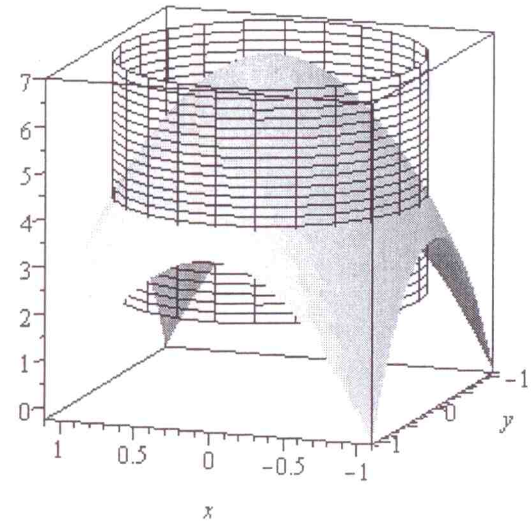
BONUS: Find the surface area from Problem II. You must show all work to get any bonus credit. (5 points)

$$= \int_0^2 u\sqrt{1+u^2}\,du = \int_1^5 \tfrac{1}{2}\sqrt{w}\,dw = \tfrac{1}{2}\cdot\tfrac{2}{3}w^{3/2}\Big|_1^5$$

$$\begin{array}{l} w=1+u^2 \\ dw=2u\,du \end{array} \qquad = \tfrac{1}{3}\left[5^{3/2}-1\right]$$

Problem III. Consider the surface area of the graph of $z = 7 - 3x^2 - 3y^2$ contained within the cylinder $1 = x^2 + y^2$. See the graph below for guidance. Please answer the following: (20 points total)

(a) Set up (DO NOT INTEGRATE) an integral in rectangular coordinates to find the surface area. (10 points)



$z_x = -6x$, $z_y = -6y$

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1+36x^2+36y^2}\, dy\, dx$$

(b) Set up (DO NOT INTEGRATE) an integral in polar coordinates to find the surface area. (10 points)

$$\int_0^{2\pi}\int_0^1 \sqrt{1+36r^2}\, r\, dr\, d\theta$$

BONUS: Find the surface area from Problem III. You must show all work to get any bonus credit. (5 points)

$u = 1+36r^2 \quad du = 72r\, dr$

$$\int_0^{2\pi}\int_1^{37} \frac{1}{72}\sqrt{u}\, du\, d\theta = \int_0^{2\pi} \frac{1}{72}\cdot\frac{2}{3}u^{3/2}\Big|_1^{37}$$

$$= \int_0^{2\pi} \frac{1}{108}\left[37^{3/2} - 1\right] d\theta = \frac{\pi}{54}\left[37^{3/2} - 1\right]$$